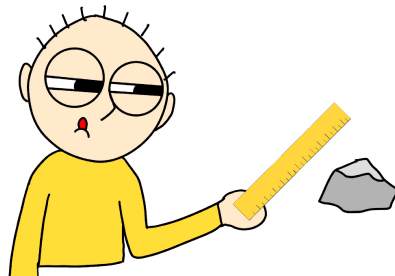




Lesson 4: Finding the Volume of Irregularly-shaped Objects

In Lesson 3, you were introduced to the measurement of volume. We said that volume was the amount of space something took up and it was a three-dimensional measurement. In Lesson 3 we learned how to find the volume of a regularly-shaped object. This was a derived measurement found by multiplying the length, width and height measurements of an object. The resulting measurement was in cubic units. Let's look now at how we might go about finding the volume of an irregularly-shaped object (one that doesn't have flat sides).

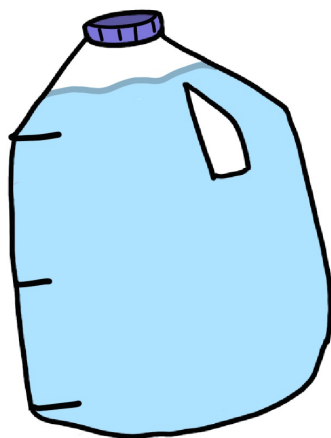


Finding the volume of an irregularly-shaped object is slightly more complicated than finding the volume of a regularly-shaped object.

Think about a rock. The surfaces of most rocks are not flat, so it's difficult to determine how many square units are present in the base of the object. And because the height of the rock can't be accurately measured, it's difficult to compute the number of cubic units that can "fit" inside the rock. Because of these irregular surfaces, we must use a different approach to find the volume of the rock.

The method we'll describe here is known as finding volume by displacement. This method will also allow us to explore the method we use to find the volume of liquids. Because finding the volume of an irregularly-shaped object requires understanding how to find the volume of a liquid, let's begin there.

Like length, the unit used to measure the volume of a liquid is a base unit. Recall that with length, our base unit using the SI system was the meter. With the volume of a liquid, our base unit using the SI system is the liter.



English units for the volume of a liquid can be in gallons, quarts or ounces. Drinks, like pop, come in bottles measured in the SI units which are liters. A commonly used part of a liter often used in science and medicine is the milliliter which is one one-thousandth of a liter.

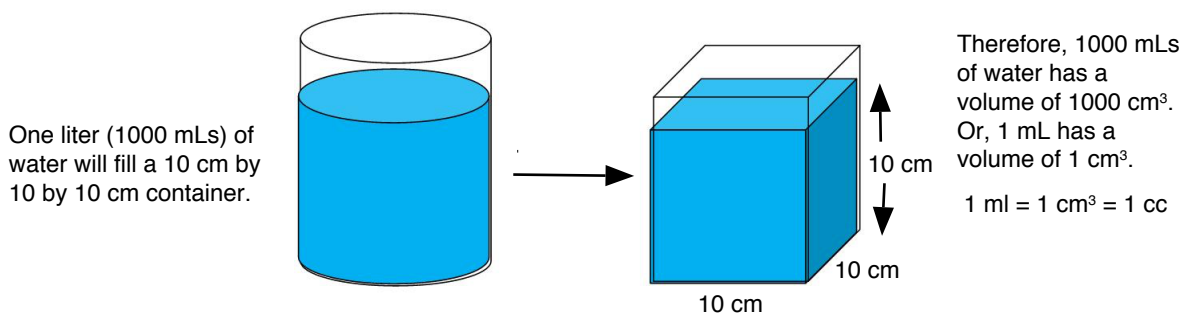
We can use the same set of prefixes as we did with length to designate multiples of liters or parts of liters. The most commonly used measure of volumes of liquids is the milliliter. Recall that the prefix milli- means one-thousandth, therefore a milliliter is equal to one-thousandth of a liter. So, if we took one liter of a liquid and divided it into 1000 equal parts, each part would represent 1 milliliter or 1 mL.

Metric Prefix Table

Prefix	Symbol	Multiplier	Exponential
tera	T	1,000,000,000,000	10^{12}
giga	G	1,000,000,000	10^9
mega	M	1,000,000	10^6
kilo	k	1,000	10^3
hecto	h	100	10^2
deca	da	10	10^1
UNIT	NONE	1	10^0
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}

Example of how to read the table: If a unit has the prefix giga (denoted by adding G before the unit symbol), that unit is 1,000,000,000 (or 10^9) times bigger than the original unit. For example, a gigawatt (GW) is 10^9 times as big as a watt.

Interestingly, if we took one liter (1000 ml) of water and placed it into a container with flat sides and found its volume using a length times width times height calculation, we would find that one liter of water has a volume of 1000 cubic centimeters. Based on this relationship, we can also say that one thousand milliliters has a volume of one thousand cubic centimeters and, therefore, one milliliter has the same volume as one cubic centimeter ($1 \text{ mL} = 1 \text{ cm}^3$ or 1 cc).



Let's go back to our discussion of finding the volume of a liquid. A measuring instrument frequently used to measure volumes of liquids is the graduated cylinder. The term graduated refers to having marks or graduations and these devices are cylindrical in shape, hence the name graduated cylinder. Look at the photo below of a graduated cylinder.



Graduated cylinders are very useful when measuring precise volumes of liquids.

Note that each mark or graduation on the cylinder represents ten milliliters of liquid. By pouring a liquid into the top opening of the cylinder, we can measure the volume of the liquid in milliliters. More precise graduated cylinders that can measure to single milliliters can be used when one needs to be more precise in measuring volumes.



Photos this page courtesy Southern Labware.

A case where this is especially important is in medicine where it is extremely important to measure correct dosages of medications. When dosages need to be carefully measured, syringes are used. Traditionally, ccs (or cubic centimeters) are used to measure liquid medications and you will commonly see dosage requirements in ccs. Knowing that a cc is the same as a milliliter can be handy with preparing medications. Other instruments commonly used in labs to measure liquids are the micropipet (my-crow pie-pet) and the buret (byoo-ret).



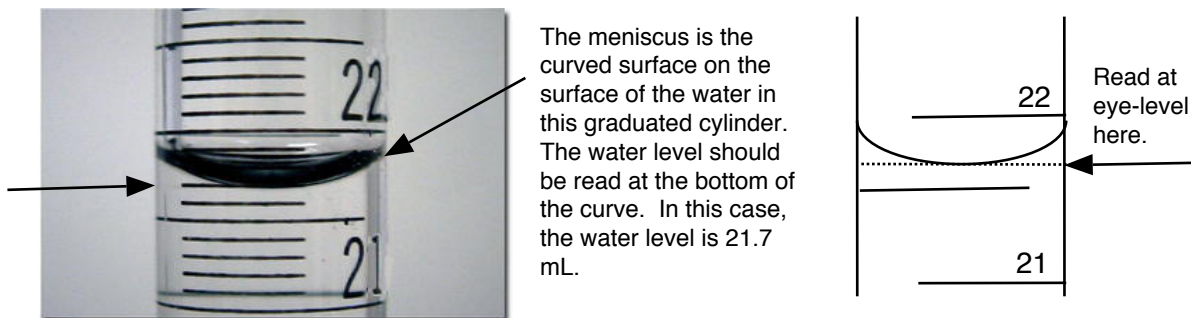
The buret (left) is a long, glass tube with a special valve at the bottom called a stopcock (orange and white in this model). Along the tube are increments of milliliters and by careful manipulation of the stopcock, a person can dispense very precise amounts of liquids.

The micropipette (right) can transfer even tinier amounts of liquids in a very precise manner. The operator depresses the thumb button on top of the handle and lowers the tip into the desired liquid. By releasing the thumb button, the exact volume of liquid is drawn up into the tip of the micropipette. Depressing the button again, allows the liquid to be dispensed in the desired location. Tiny, exact portions of milliliters of liquids can be measured using these instruments.



Let's practice making some liquid volume measurements by examining some diagrams of graduated cylinders with varying volumes of liquid. Take a look at the first graduated cylinder.

Note that the upper surface of the liquid is not flat, but instead, curved. The curvature is due to the fact that liquids experience an attractive force to the side of the cylinder which results in a curved surface. This attractive force is known as adhesion. This curved shape is known as the meniscus.

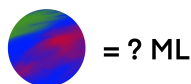


Because of this curve, it may be slightly confusing to know exactly where to measure the volume of liquid. The accepted technique is to use the “bottom” of the meniscus as the point to make your measurement. Notice in the diagram how the “bottom” of the meniscus is used to measure the volume of a liquid inside a graduated cylinder.

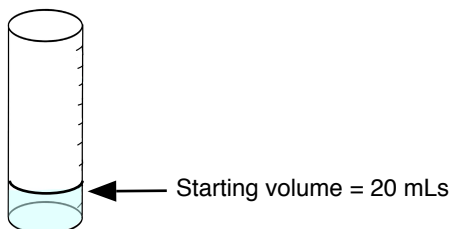
Now that we know how to measure the volume of a liquid, let's go back to our discussion of measuring the volume of an irregularly-shaped object. We said earlier that we can't utilize our length x width x height method because the surfaces of an irregularly-shaped object are not flat. We have parts of the object that protrude or are sunken on the surface. So, how can we find the volume?

We'll find the volume by using the displacement method. The term displacement means that something is moved “out of the way” due to the action of another object or substance. In this case we'll allow our irregularly-shaped object (our rock) to displace another substance. A readily available substance we can use is water.

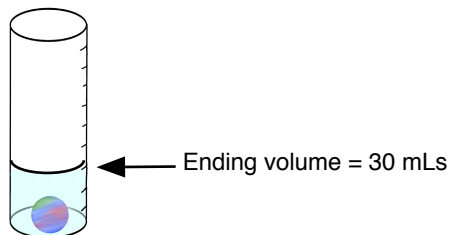
Now before we find the volume of our rock, let's look at a simpler example. Let's pretend we need to find the volume of a marble and we will do so using the displacement method.



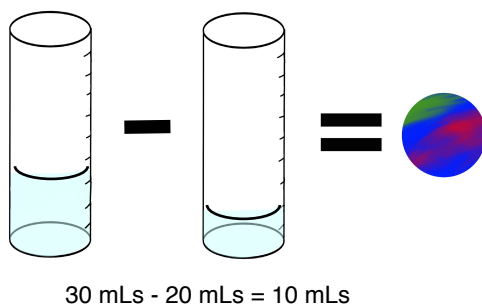
We'll begin by partially filling our graduated cylinder with water. We'll call this amount of water our "starting volume." As you can see in the photo below, our starting volume will be 20 mL.



Next, we'll drop in our marble.

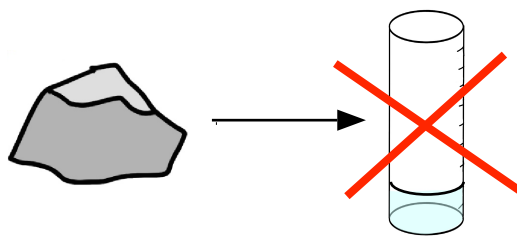


What happens to the water level? It goes up, right? Why does that happen? Obviously, the marble, which has a volume of its own, pushes the water out of the way. It displaces the water. How much water was displaced? The amount (volume) of water that gets displaced is equal to the volume of the object that did the displacing. In other words, the volume *change* of the water equals the volume of marble.



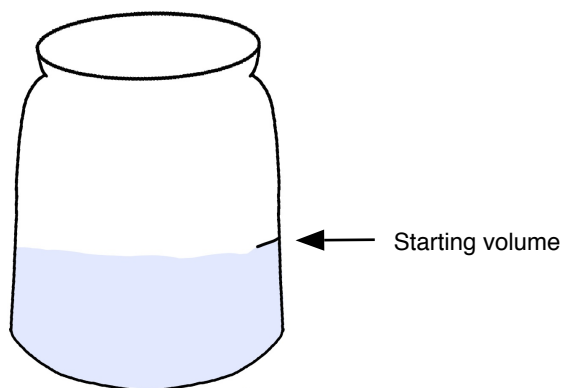
By subtracting the starting volume from the ending volume, we can find the amount of water which was displaced. In our example our ending volume was 30 mL. Therefore the volume of water that was displaced is $30 \text{ mLs} - 20 \text{ mLs} = 10 \text{ mLs}$. This means the volume of our marble is 10 mLs.

But what about an irregularly-shaped object that won't fit down inside a graduated cylinder?

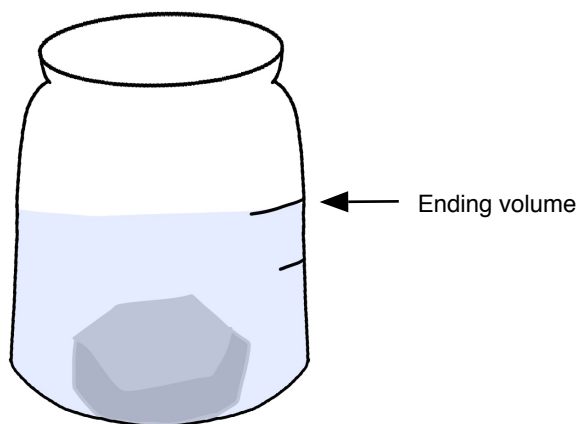


Think back to our earlier question about finding the volume of a rock. What if our rock won't fit down inside a graduated cylinder? Can you think of how you might go about measuring its volume? Because the rock won't fit into our graduated cylinder, we'll use another container to hold our "starting" volume of water. We'll then add the rock to the water in that container and determine the change in volume.

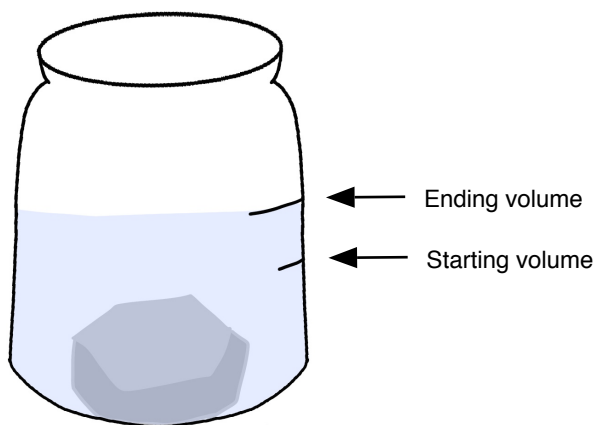
Here you can see that we've taken a jar and partially filled it with water. We'll mark the side of the jar at the level of the water.



In this diagram you can see that we've added our rock to the container. Note how the water was indeed displaced as we expected. As we discussed earlier, this change in volume is equal to the volume of our rock. We'll place a second mark now at this "new" level. It's the difference between our two marks that we'll need to find. This mark is our "ending" mark.



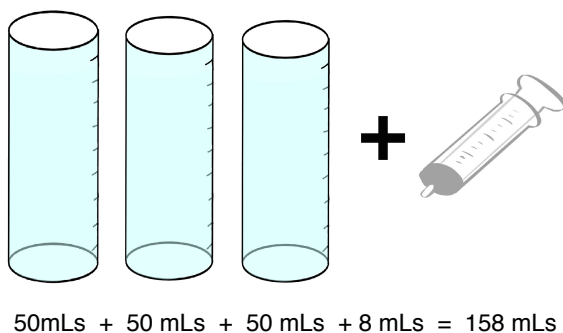
We'll find this difference by first removing our rock and all of the water from the jar. We'll refill the jar to our first mark (the starting mark) and then by adding known amounts of water, measured using our graduated cylinder, we can find how much water it takes to get to the "ending" mark. The difference in volume between the starting and ending volumes will equal the volume of the rock.



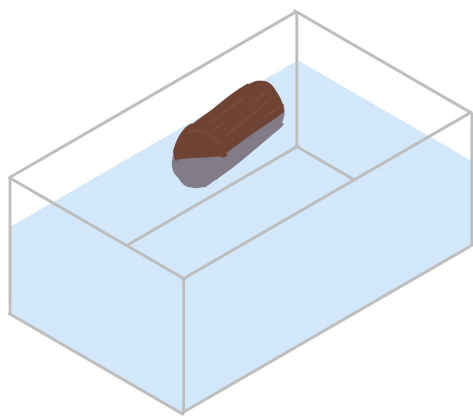
To help us not accidentally overfill water past the ending mark, we'll use a syringe to slowly come up to the ending mark. Recall that most syringes measure in ccs which are the same as milliliters. While filling, it's important to keep track of how many times you fill the graduated

cylinder or syringe. When you've filled the jar to the ending mark, the total number of milliliters you used is equal to the volume of the rock (or unknown object).

In our example here, you can see that we filled our graduated cylinder three times, each cylinder holding 50 mLs for a total of 150 mLs. We then "topped it off" by adding 8 ccs (mLs) with the syringe. This gave us a total of 158 mLs displacement which means the rock has a volume of 158 mLs. How many cubic centimeters would this be equal to? If you said, 158 ccs, you're correct!



Note that finding the volume of irregularly-shaped objects by displacement works best for objects that sink when placed into water. What about something like a piece of wood that tends to float when placed in water?



Finding the volume of an irregularly-shaped object which floats when placed into water can be challenging!

Can you think of any ideas on how to accomplish this? Could you possibly push it down beneath the surface of the water or maybe place a heavy object on top of it and force it down beneath the surface? Will you need to find the volume of heavy object, too? What if you tied a heavy object to your unknown object and allowed it to pull the object down below the water level? Would you

have to take into account the volume of the rope or string you used for tying, too? With some creative thinking, you should be able to find the volume of just about any solid object using the displacement method.

Before we leave this lesson on volume, let's look at the units used for volume in the English system. You are likely familiar with gallons, quarts and pints. A quart is one-fourth of a gallon while a pint is one-half of a quart. A pint is made of two cups. Cups can be divided into eight ounces. Two tablespoons make an ounce and three teaspoons make a tablespoon. As you can see, while Americans continue to use the English system, the simplicity of the metric system for measuring volume is appealing.

1 gallon = 4 quarts
 1 quart = 2 pints
 1 pint = 2 cups
 1 cup = 8 ounces
 1 ounce = 2 Tablespoons
 1 Tablespoon = 3 teaspoons



Lab 1: In this lab you'll practice finding the volume of four different irregularly-shaped objects.

Materials to gather: 4 irregularly-shaped objects, jar or other container which can hold water, measuring instrument such as a graduated cylinder, syringe or pipette. A soda straw can work, too, when marked at specific increments.

Procedure: Find the volume of each object according to the procedure presented in your lesson. Record your results and then have your mom, dad or teacher check your accuracy in making these measurements.

Challenge 1: In this challenge you will pretend to be a company which packages chicken eggs. Because packaging materials are relatively expensive for your company, you'll want to use the minimum amount of materials, yet have the egg fully enclosed on all surfaces. Choose one chicken egg and build a container that will fully enclose the egg yet only provide a maximum of clearance of 2 mm on any surface between the egg and the wall of the container. In other words, when the egg is placed into the container, you should see no more than 2mm of space between the surface of the egg and the interior surface of the box. The box should have a lid, working hinges and a locking mechanism. You may boil your egg first if you feel like it would be easier to handle as you construct and then test your box. Hint: the interior surface of the box need not be the box itself, but another "surface".

